Receding Horizon Control for Hot Strip Mill Cooling Systems

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***Abstract*—In a hot strip mill, the strip is cooled by spraying water from the top and bottom on the runout table (ROT) before the strip is coiled. The desired mechanical properties and metallurgical structure of the strip are achieved by controlling the cooling rate and temperature of the strip on the ROT. In this paper, we propose a design method of receding horizon control for controlling the temperature of a strip whose mathematical model is described by a nonlinear partial differential equation (PDE). We provide a fast numerical solution method for solving the nonlinear optimization problem for a class of nonlinear PDEs. Moreover, a state observer with an unscented Kalman filter for estimating the inhomogeneously distributed temperature of the strip is incorporated into the receding horizon controller. The effectiveness of the proposed method is verified by numerical simulation.**

***Index Terms*—Nonlinear system, optimization algorithm, partial differential equation, receding horizon control.**

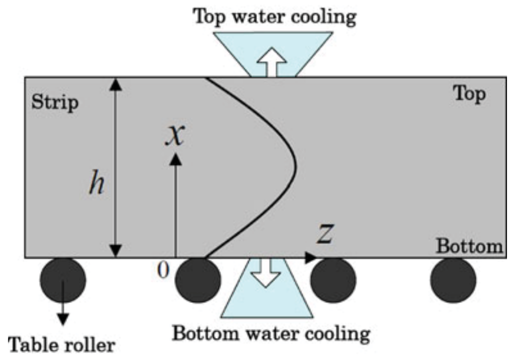

Fig. 1. Hot strip mill cooling system.

I. Introduction

HOT ROLLING is an essential industrial process in the production of sheet steels, a widely used product in manufacturing and construction [1], [2]. In a hot strip mill, the strip is cooled by spraying water from the top and bottom on the runout table (ROT) before the strip is coiled. The desired mechanical properties and metallurgical structure of the strip are achieved by controlling the cooling rate and temperature of the strip on the ROT. Therefore, it is important to carefully control the temperature profile in the cooling process to regulate the quality of the steel. Temperature control of the hot-rolled strip has been of great interest in the steel/metal industry [3]–[6]. Fig. 1 shows the layout of a strip on the ROT.

A number of control methods for the cooling process rely on either lookup tables or a direct empirical relation between the water flow and strip temperature [7]–[9]. However, these control methods are only valid for a particular hot strip mill with a restricted range of system parameters. In the case of variation in the system parameters, feedback action is necessary to adapt the controller to take into account the changed conditions. Several control methods that incorporate both feedforward and feedback action to control the temperature of the strip on the ROT have been proposed [10]–[14]. However, the control methods in [10]–[12] use an approximate model of the cooling process to predict the temperature of the strip and iteratively estimate the amount of water flow required to achieve the desired temperature of the strip, although the actual model of the cooling process is a nonlinear partial differential equation (PDE). Moreover, although a nonlinear PDE model for the temperature of the strip has been addressed [13], [14], the optimality of the controller has not yet been discussed. In this paper, therefore, we propose an optimal control method for a nonlinear PDE model of the temperature of the strip to achieve the desired temperature profile of the strip. The proposed method enables us to design a controller that takes the efficiency of water consumption into account.

Receding horizon control is a kind of optimal feedback control, and its performance index has a moving initial time and a moving terminal time [15], [16]. A fast algorithm called the continuation/generalized minimum residual (C/GMRES) method has been proposed [17] to solve the nonlinear receding horizon control problem for systems described by ordinary differential equations. However, a design method of receding horizon control for nonlinear PDEs has not yet been proposed. In fact, appropriate reformulation and modification are necessary to apply receding horizon control with the C/GMRES method to systems described by nonlinear PDEs. The objective of this paper is to develop a design method of receding horizon control for nonlinear PDEs and to show that such a method can be effectively applied to the control of the cooling process of a strip on a ROT. Moreover, we also propose an estimation method for the inhomogeneously distributed temperature of the strip. State

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estimation for nonlinear PDEs is an open problem and beyond the scope of this paper. Nevertheless, we design an observer with an unscented Kalman filter (UKF) [18] using the discretized nonlinear model of the temperature of the strip for the state estimation problem.

This paper is organized as follows. The system considered here is defined in Section II. Section III plays an essential role in this study. In Section III, we formulate the receding horizon control problem and derive the stationary conditions that must be satisfied for a performance index to be optimized. Section IV is devoted to a description of the numerical algorithm for solving the discretized stationary conditions. In Section V, we consider the state estimation problem for the temperature of the strip using an observer with a UKF. In Section VI, a numerical simulation is conducted to verify the effectiveness of the proposed method. Finally, some concluding remarks are given in Section VII.

II. System Description

Let $\mathbb{R}$ and $\mathbb{R}_+$ denote the sets of real numbers and nonnegative real numbers, respectively. Let $\mathbb{N}_+$ denote the set of positive integers. The transpose of $\mathbf{A} \in \mathbb{R}^{n\times n}$ is denoted by $\mathbf{A}'$. Let $\mathbf{I}$ denote the identity matrix. Let $\mathrm{sgn}(x)$ denote the signum function. The system parameters used in this paper are listed in Table I. Let Ω and $\partial\Omega$ be sets defined by $\Omega := \{x|0 \le x \le h\}$ and $\partial\Omega := \{x|x = 0, x = h\}$, respectively. We assume that the nonlinear functions $c(z)$, $b(z)$, $H(z)$, and $\gamma(z)$ are all continuous and differentiable functions. As a prerequisite for control, the cooling process on the ROT has been modeled in [3]. The equation that governs the cooling process on the ROT is a nonlinear heat conduction equation

$$\rho c\,(z)\,\frac{\partial z(x,t)}{\partial t} = \frac{\partial}{\partial x}\left(b\,(z)\,\frac{\partial z(x,t)}{\partial x}\right) + H(z)\gamma(z) \quad (1a)$$

with the initial condition $z(x,t) = z_0(x)$ at $t = 0$. Taking into account the fact that the strip can be cooled not only by water but also by air, we consider the following boundary conditions:

$$\frac{\partial z(x,t)}{\partial x} = k(z,u,z_w,z_a) \quad \text{at } x = 0 \quad (1b)$$

$$\frac{\partial z(x,t)}{\partial x} = -k(z,u,z_w,z_a) \quad \text{at } x = h \quad (1c)$$

where $k(z,u,z_w,z_a) \in \mathbb{R}_+$ denotes the heat transfer function caused by the water and air. We assume that z_w and z_a are known constants. Thus, the arguments z_w and z_a of k are omitted hereafter. We assume that $k(z,u)$ is a continuous and differentiable function with respect to z and u. The function $k(z,u)$ has to be determined empirically so that the system model can be matched with the actual cooling process. The specific model of $k(z,u)$ depends on the hot strip mill cooling system under consideration. In this section, we consider the boundary conditions in a general setting as (1b) and (1c) without referring to any specific cooling system on the ROT. Thus, the general design philosophy discussed herein can be applied to any specific processes. Nevertheless, particular details pertaining to a specific process are discussed in Section VI.

TABLE I
System Parameters

t	time [s]
x	coordinate along thickness direction [m]
$z(x,t)$	strip temperature [°C]
z_a	temperature of air [°C]
$\omega(t)$	water flow rate [m^3/s]
z_w	temperature of water [°C]
$u(\omega, z_w)$	control variable
ρ	density of strip material [kg/m^3]
h	thickness of strip [m]
$c(z)$	specific heat of strip material [J/kg°C]
$b(z)$	thermal conductivity of strip material [W/m°C]
$H(z)$	latent heat of phase transformation [J/kg]
$\gamma(z)$	rate of phase transformation [kg/m^{3}s]
$d(z)$	specific enthalpy [J/kg°C]
z_f	desired temperature [°C]
T	evaluation interval [s]
σ	Stefan-Boltzmann constant [kg/s^3 °C^4]
ε	emissivity of steel
$k(z,u,z_w,z_a)$	heat transfer function
p,q,r,s	weighting constants

It was shown in [14] that the product of the latent heat of the phase transformation $H(z)$ and the rate of the phase transformation $\gamma(z)$ can be regarded as the time derivative of the specific enthalpy $d(z)$. Substituting

$$H(z)\gamma(z) = \frac{\partial d(z)}{\partial z}\frac{\partial z}{\partial t} \quad (2)$$

into (1a), we obtain

$$\left\{\rho c(z) - \frac{\partial d(z)}{\partial z}\right\}\frac{\partial z(x,t)}{\partial t} = \frac{\partial}{\partial x}\left(b\,(z)\,\frac{\partial z(x,t)}{\partial x}\right). \quad (3)$$

Let $a(z)$ be defined by $a(z) := \rho c(z) - \partial d(z)/\partial z$. We assume that $a(z) \ne 0$ and $b(z) \ne 0$ for all z. In the subsequent discussion, we consider the following system:

$$\frac{\partial z(x,t)}{\partial t} = \mathcal{F}\,(z) \quad \text{for } x \in \Omega \quad (4a)$$

$$\frac{\partial z(x,t)}{\partial x} = \alpha(x)k(z,u) \quad \text{for } x \in \partial\Omega \quad (4b)$$

where $\mathcal{F}(z) : \mathbb{R} \to \mathbb{R}$ and $\alpha(x) : \partial\Omega \to \{-1, 1\}$ are defined by

$$\mathcal{F} := \frac{1}{a(z)}\left\{\frac{\partial}{\partial x}\left(b\,(z)\,\frac{\partial z(x,t)}{\partial x}\right)\right\}$$

$$\alpha(x) := \mathrm{sgn}\left(\frac{h}{2} - x\right).$$

Considering that a cooling system has an upper bound of the water rate, we impose an inequality constraint on u as

$$0 \le u(t) \le u_{\max}, \quad \forall t \in \mathbb{R}_+ \quad (5)$$

where $u_{\max}$ is a positive constant. Then, it follows from (5) that

$$\left|u(t) - \frac{u_{\max}}{2}\right| \le \frac{u_{\max}}{2}. \quad (6)$$

This inequality constraint can be converted into an equality constraint by introducing a slack variable $v(t) : \mathbb{R}_+ \to \mathbb{R}$ as follows:

$$C[u,v] := \frac{1}{2}\left\{\left(u - \frac{u_{\max}}{2}\right)^2 - \left(\frac{u_{\max}}{2}\right)^2 + v^2\right\} = 0. \quad (7)$$

Note that the slack variable v is squared to constrain the sign of $\{(u-u_{\max}/2)^2-(u_{\max}/2)^2\}$ to be nonpositive. Since the sign of v does not affect the optimality, the optimal solution can bifurcate and an algorithm cannot determine the update of v when $v=0$. To avoid the singularity at $v=0$, a small dummy penalty is also added to the last term in the performance index.

III. Receding Horizon Control

In this section, the nonlinear optimal control problem for system (4) is discussed. The control input at each time t is determined so as to minimize the performance index given by

$$\bar{J}=\phi\left[z(x,t+T)\right]+\int_t^{t+T} L[z(x,\tau),u(\tau),v(\tau)]d\tau$$
$$\phi:=\frac{1}{2}\int_\Omega p\{z(x,t+T)-z_f(t+T)\}^2dx$$
$$L:=\frac{1}{2}\left(\int_\Omega q\{z(x,\tau)-z_f(\tau)\}^2dx+ru^2(\tau)-2sv(\tau)\right) \quad (8)$$

where $z_f(t)\in\mathbb{R}$ is the desired homogeneous temperature profile, $T\in\mathbb{R}_+$ is the evaluation interval of the performance index, and $p,q,r,$ and $s\in\mathbb{R}_+\backslash\{0\}$ are weighting constants. The optimization problem of (8) subject to constraints (4) and (7) can be reduced to minimizing the following performance index introduced by using the costate $\lambda(x,t):\Omega\times\mathbb{R}_+\to\mathbb{R}$ and the Lagrange multiplier $\mu(t):\mathbb{R}_+\to\mathbb{R}$ associated with the equality constraint

$$J=\phi\left[z(x,t+T)\right]+\int_t^{t+T}\left(H[z,u,v,\lambda,\mu]+Q[z,\lambda]\right)d\tau \quad (9)$$

where H and Q are defined by

$$H:=L+\int_\Omega \lambda(x,\tau)\mathcal{F}(z)dx+\mu(\tau)C[u,v]$$
$$Q:=\int_\Omega -\lambda(x,\tau)\frac{\partial z(x,\tau)}{\partial\tau}dx.$$

Let z_x, z_{xx}, and b_z denote $\partial z/\partial x$, $\partial^2 z/\partial x^2$, and $\partial b/\partial z$, respectively. Now, observing that

$$\mathcal{F}=\frac{1}{a(z)}\left\{b_z(z)z_x^2+b(z)z_{xx}\right\} \quad (10)$$

it follows that

$$\frac{\partial\mathcal{F}}{\partial z}=\frac{\partial}{\partial z}\left\{\frac{b_z(z)}{a(z)}\right\}z_x^2+\frac{\partial}{\partial z}\left(\frac{b(z)}{a(z)}\right)z_{xx}$$
$$\frac{\partial\mathcal{F}}{\partial z_x}=\left\{\frac{2b_z(z)}{a(z)}\right\}z_x$$
$$\frac{\partial\mathcal{F}}{\partial z_{xx}}=\left\{\frac{b(z)}{a(z)}\right\}.$$

Let δJ, δz, δz_x, δz_{xx}, $\delta\lambda$, δu, δv, and $\delta\mu$ denote the variations (infinitesimal changes) in J, z, z_x, z_{xx}, λ, u, v, and μ, respectively. It is important to note that we can apply integration by parts to the computation of δJ as follows:

$$\int_\Omega\frac{\partial H}{\partial z_x}\delta z_x dx=\int_\Omega\left(\frac{\partial H}{\partial z_x}\right)\frac{\partial\delta z}{\partial x}dx$$
$$=\left[\left(\frac{\partial H}{\partial z_x}\right)\delta z\right]_0^h-\int_\Omega\frac{\partial}{\partial x}\left(\frac{\partial H}{\partial z_x}\right)\delta z dx \quad (11)$$

$$\int_\Omega\frac{\partial H}{\partial z_{xx}}\delta z_{xx}dx=\int_\Omega\left(\frac{\partial H}{\partial z_{xx}}\right)\frac{\partial^2\delta z}{\partial x^2}dx$$
$$=\left[\left(\frac{\partial H}{\partial z_{xx}}\right)\frac{\partial\delta z}{\partial x}\right]_0^h-\left[\frac{\partial}{\partial x}\left(\frac{\partial H}{\partial z_{xx}}\right)\delta z\right]_0^h$$
$$+\int_\Omega\frac{\partial^2}{\partial x^2}\left(\frac{\partial H}{\partial z_{xx}}\right)\delta z dx. \quad (12)$$

Considering that, from boundary condition (4b), we obtain

$$\frac{\partial\delta z}{\partial x}=\alpha(x)\left(\frac{\partial k}{\partial z}\delta z+\frac{\partial k}{\partial u}\delta u\right)\quad \text{for } x\in\partial\Omega. \quad (13)$$

We can substitute (13) into $(\partial\delta z/\partial x)$ in (12) for $x=0$ and $x=h$. It is also important to note that we have the following integration by parts:

$$\int_t^{t+T}\left(Q[z+\delta z]-Q[z]\right)d\tau=\int_t^{t+T}\int_\Omega-\lambda\frac{\partial\delta z}{\partial\tau}dxd\tau$$
$$=\left[\int_\Omega-\lambda(x,\tau)\delta z(x,\tau)dx\right]_t^{t+T}+\int_t^{t+T}\int_\Omega\frac{\partial\lambda}{\partial\tau}\delta z dxd\tau$$
$$=\int_\Omega-\lambda(x,t+T)\delta z(x,t+T)dx+\int_t^{t+T}\int_\Omega\frac{\partial\lambda}{\partial\tau}\delta z dxd\tau. \quad (14)$$

Note that we take $\delta z(x,t)=0$ in (14), because $z(x,\tau)$ is fixed at $\tau=t$. Taking (11)–(14) into account, we obtain the variation in J as follows:

$$\delta J=\int_t^{t+T}\left[\int_\Omega\left\{\mathcal{F}(z)-\frac{\partial z}{\partial\tau}\right\}\delta\lambda(x,\tau)dx+\int_\Omega\left\{\frac{\partial\lambda}{\partial\tau}\right.\right.$$
$$\left.+\frac{\partial H}{\partial z}-\frac{\partial}{\partial x}\left(\frac{\partial H}{\partial z_x}\right)+\frac{\partial^2}{\partial x^2}\left(\frac{\partial H}{\partial z_{xx}}\right)\right\}\delta z(x,\tau)dx$$
$$+\left[\left\{\frac{\partial H}{\partial z_x}-\frac{\partial}{\partial x}\left(\frac{\partial H}{\partial z_{xx}}\right)+\alpha(x)\left(\frac{\partial H}{\partial z_{xx}}\right)\left(\frac{\partial k}{\partial z}\right)\right\}\delta z(x,\tau)\right]_0^h$$
$$+\left\{\frac{\partial H}{\partial u}-\sum_{x\in\partial\Omega}\left(\frac{\partial H}{\partial z_{xx}}\right)\left(\frac{\partial k}{\partial u}\right)\right\}\delta u(\tau)+\frac{\partial H}{\partial v}\delta v(\tau)$$
$$\left.+\frac{\partial H}{\partial\mu}\delta\mu(\tau)\right]d\tau+\int_\Omega\left(p\{z(x,t+T)-z_f(t+T)\}\right.$$
$$\left.+\lambda(x,t+T)\right)\delta z(x,t+T)dx.$$

Let k_z, k_u, and C_u denote $\partial k/\partial z$, $\partial k/\partial u$, and $\partial C/\partial u$, respectively. Consequently, we obtain the necessary conditions for a stationary value of J over the horizon $(t\le\tau\le t+T)$ as follows. For $x\in\Omega$, we have

$$\frac{\partial z}{\partial\tau}=\mathcal{F}(z) \quad (15a)$$
$$p\{z(x,t+T)-z_f(t+T)\}=\lambda(x,t+T) \quad (15b)$$

$$\frac{\partial \lambda}{\partial \tau} = -\frac{\partial H}{\partial z} + \frac{\partial}{\partial x}\left(\frac{\partial H}{\partial z_x}\right) - \frac{\partial^2}{\partial x^2}\left(\frac{\partial H}{\partial z_{xx}}\right) \tag{15c}$$

and, for $x \in \partial\Omega$, we have

$$\frac{\partial H}{\partial z_x} - \frac{\partial}{\partial x}\left(\frac{\partial H}{\partial z_{xx}}\right) + \alpha(x)\left(\frac{\lambda b}{a}\right) k_z = 0 \tag{15d}$$

$$ru + \mu C_u - \sum_{x \in \partial\Omega} \left(\frac{\lambda b}{a}\right) k_u = 0 \tag{15e}$$

$$\mu v - s = 0 \tag{15f}$$

$$C[u, v] = 0. \tag{15g}$$

A well-known difficulty of nonlinear optimal control is that the obtained stationary conditions cannot be solved analytically in general. To solve the above stationary conditions (15a)–(15g) by a numerical solution method, we must discretize these equations into finite difference equations.

Here, we divide the space and time into $M \in \mathbb{N}_+$ grids and $N \in \mathbb{N}_+$ grids, respectively. Hence, each division width is given by $\Delta x := h/(M-1)$ and $\Delta \tau := T/(N-1)$. By means of the discretization, $z(x, \tau)$ $(0 \le x \le h, t \le \tau \le t+T)$ can be described by $z_{i,j}(t)$ $(i = 1, \ldots, M, j = 1, \ldots, N)$, where the subscripts i and j denote space and time, respectively. For other variables, we adopt such notation without explanation. Let $\mathbf{z}_j(t) \in \mathbb{R}^M$ and $\lambda_j(t) \in \mathbb{R}^M$ denote $\mathbf{z}_j(t) := [z_{1,j}(t), \ldots, z_{M,j}(t)]'$ and $\lambda_j(t) := [\lambda_{1,j}(t), \ldots, \lambda_{M,j}(t)]'$, respectively. Let $\mathbf{z}_{fN}$ be defined by $\mathbf{z}_{fN} := [z_f(t+T), \ldots, z_f(t+T)]' \in \mathbb{R}^M$. The Crank–Nicolson method [19] is a finite difference method used for numerically solving PDEs. It is a second-order method in time and space, and is numerically stable. Adopting the finite difference method, we obtain the approximately discretized stationary conditions over the horizon $(t \le \tau \le t+T)$ as follows. For $x_i \in \Omega$, we have

$$\mathbf{A}(\mathbf{z}_j)\mathbf{z}_{j+1} = \mathbf{B}(\mathbf{z}_j, u_j) \tag{16a}$$

$$p(\mathbf{z}_N - \mathbf{z}_{fN}) = \lambda_N \tag{16b}$$

$$\mathbf{C}(\lambda_{j+1})\lambda_j = \mathbf{D}(\lambda_{j+1}, \mathbf{z}_{j+1}, u_{j+1}) \tag{16c}$$

and, for $x_i \in \partial\Omega$, we have

$$ru_j + \mu_j C_u(u_j) - \sum_{i \in \{1, M\}} \left(\frac{\lambda_{i,j} b(z_{i,j})}{a(z_{i,j})}\right) k_u(z_{i,j}, u_j) = 0 \tag{16d}$$

$$\mu_j v_j - s = 0 \tag{16e}$$

$$C[u_j, v_j] = 0 \tag{16f}$$

where $\mathbf{A}, \mathbf{C} \in \mathbb{R}^{M \times M}$ and $\mathbf{B}, \mathbf{D} \in \mathbb{R}^M$ denote matrix functions and vector functions, respectively. Note that boundary condition (4b) is also discretized and employed in (16a). Also, note that the equations obtained by discretizing (15c) and (15d) are unified into (16c). Moreover, we assume that $\mathbf{A}$ and $\mathbf{C}$ are invertible and guarantee the stability of (16a) and (16c), respectively.

IV. Numerical Solution Algorithm

Once we have the finite difference equations for the optimization conditions, we can find the optimal control input using a numerical solution algorithm. An iterative optimization method such as Newton's method can be executed to solve the optimization problem. However, an iterative optimization method is computationally expensive and is not suitable for systems controlled with a brief sampling period. On the one hand, a fast algorithm, called the C/GMRES method, for achieving real-time optimization without offline computation has been proposed [17]. In general, the optimal solution varies smoothly with respect to time. Then, using the C/GMRES method [17], we can trace the optimal solution, without the need for iterative methods, by integrating a differential equation for updating the control sequence. Here, we adopt the C/GMRES method to solve the obtained stationary conditions, because the C/GMRES algorithm is not only faster but also more numerically robust than the conventional algorithm. Detailed features of the C/GMRES method compared with other methods can be found in [17]. For the sake of completeness, a brief description of the C/GMRES method applied to this problem is presented in the subsequent discussion.

Let $\mathbf{U}(t) \in \mathbb{R}^{3N}$ be defined as follows:

$$\mathbf{U}(t) := [\, u_1(t), v_1(t), \mu_1(t), \ldots, u_N(t), v_N(t), \mu_N(t) \,]'. \tag{17}$$

For a given initial state $\mathbf{z}_1(t)$, $\{\mathbf{z}_j(t)\}_{j=1}^N$ is calculated recursively using (16a). Then, for the terminal costate $\lambda_N(t)$ determined from (16b), $\{\lambda_j(t)\}_{j=1}^N$ is also calculated recursively from $j = N$ to $j = 1$ using (16c). Since $\{\mathbf{z}_j(t)\}_{j=1}^N$ and $\{\lambda_j(t)\}_{j=1}^N$ are determined by $\mathbf{z}_1(t)$ and $\mathbf{U}(t)$ through (16a)–(16c), (16d)–(16f) can be regarded as one equation written as

$$\mathbf{K}(\mathbf{U}(t), \mathbf{z}_1(t), t) := \begin{bmatrix} \mathbf{K}_1 \\ \vdots \\ \mathbf{K}_N \end{bmatrix} = 0 \tag{18}$$

where

$$\mathbf{K}_j := \begin{bmatrix} ru_j + \mu_j C_u(u_j) - \sum_{i \in \{1, M\}} \frac{\lambda_{i,j} b(z_{i,j})}{a(z_{i,j})} k_u(z_{i,j}, u_j) \\ \mu_j v_j - s \\ C[u_j, v_j] \end{bmatrix}.$$

If (18) is solved with respect to $\mathbf{U}(t)$ for a given $\mathbf{z}_1(t)$ at each time t, then the optimal control input can be determined. We can evaluate $\|\mathbf{K}\|$ for the optimality performance, because the optimal solution must satisfy $\|\mathbf{K}\| = 0$. Instead of solving $\mathbf{K}(t) = 0$ itself at each time by such an iterative method as Newton's method, we find the derivative of $\mathbf{U}(t)$ with respect to time such that $\mathbf{K}(t) = 0$ is satisfied identically. Namely, we determine $\dot{\mathbf{U}}(t)$ so that

$$\dot{\mathbf{K}}(t) = \mathbf{A}_s \mathbf{K}(t) \tag{19}$$

where $\mathbf{A}_s$ is a stable matrix introduced to stabilize $\mathbf{K} = 0$. $\mathbf{A}_s$ is a design parameter to determine the convergence rate of $\|\mathbf{K}(t)\|$. It was shown in [17] that if we choose $\mathbf{A}_s = -\mathbf{I}/\Delta t$ (Δt: sampling period), then the stability of forward difference equation is guaranteed. Also, we can empirically adjust $\mathbf{A}_s$ to achieve a better performance.

By total differentiation, we obtain

$$\frac{\partial \mathbf{K}}{\partial \mathbf{U}} \dot{\mathbf{U}}(t) = \mathbf{A}_s \mathbf{K} - \frac{\partial \mathbf{K}}{\partial \mathbf{z}_1} \dot{\mathbf{z}}_1 - \frac{\partial \mathbf{K}}{\partial t} \tag{20}$$

which can be regarded as a linear algebraic equation with a coefficient matrix $(\partial \mathbf{K}/\partial \mathbf{U})$, to determine $\dot{\mathbf{U}}$ for given $\mathbf{U}$, $\mathbf{z}_1$, $\dot{\mathbf{z}}_1$, and t. Then, if the Jacobian $(\partial \mathbf{K}/\partial \mathbf{U})$ is nonsingular, we obtain a differential equation for $\mathbf{U}(t)$ as

$$\dot{\mathbf{U}}(t) = \left(\frac{\partial \mathbf{K}}{\partial \mathbf{U}}\right)^{-1} \left(\mathbf{A}_s \mathbf{K} - \frac{\partial \mathbf{K}}{\partial \mathbf{z}_1}\dot{\mathbf{z}}_1(t) - \frac{\partial \mathbf{K}}{\partial t}\right). \tag{21}$$

We can update the solution $\mathbf{U}(t)$ of $\mathbf{K}(\mathbf{U}(t), \mathbf{z}_1(t), t) = 0$, without the need for iterative optimization methods, by integrating (21) in real time as, for example, $\mathbf{U}(t+\Delta t) = \mathbf{U}(t) + \dot{\mathbf{U}}(t)\Delta t$, where Δt denotes the sampling period. This approach is a kind of continuation method [20] in the sense that the solution curve $\mathbf{U}(t)$ is traced by integrating a differential equation.

From the computational point of view, the differential equation (21) still involves expensive operations, that is, Jacobians $(\partial \mathbf{K}/\partial \mathbf{U})$, $(\partial \mathbf{K}/\partial \mathbf{z}_1)$, and $(\partial \mathbf{K}/\partial t)$ and the linear algebraic equation associated with $(\partial \mathbf{K}/\partial \mathbf{U})^{-1}$. To reduce the computational cost in the Jacobians and the linear equation, we employ two techniques, that is, the forward difference approximation for products of Jacobians and vectors, and the GMRES method [21] for the linear algebraic equation.

First, we approximate the products of the Jacobians and some $\mathbf{w}_1 \in \mathbb{R}^{3N}$, $\mathbf{w}_2 \in \mathbb{R}^{M}$, and $w_3 \in \mathbb{R}$ with the forward difference as follows:

$$\begin{aligned} &\frac{\partial \mathbf{K}(\mathbf{U}, \mathbf{z}_1, t)}{\partial \mathbf{U}}\mathbf{w}_1 + \frac{\partial \mathbf{K}(\mathbf{U}, \mathbf{z}_1, t)}{\partial \mathbf{z}_1}\mathbf{w}_2 + \frac{\partial \mathbf{K}(\mathbf{U}, \mathbf{z}_1, t)}{\partial t}w_3 \\ &\simeq D_s\mathbf{K}(\mathbf{U}, \mathbf{z}_1, t : \mathbf{w}_1, \mathbf{w}_2, w_3) \\ &:= \frac{[\mathbf{K}(\mathbf{U} + s\mathbf{w}_1, \mathbf{z}_1 + s\mathbf{w}_2, t + sw_3) - \mathbf{K}(\mathbf{U}, \mathbf{z}_1, t)]}{s} \end{aligned}$$

where s is a positive real number. Then, (19) is approximated as

$$D_s\mathbf{K}(\mathbf{U}, \mathbf{z}_1, t : \dot{\mathbf{U}}, \dot{\mathbf{z}}_1, 1) = \mathbf{A}_s\mathbf{K}(\mathbf{U}, \mathbf{z}_1, t)$$

which is equivalent to

$$D_s\mathbf{K}(\mathbf{U}, \mathbf{z}_1 + s\dot{\mathbf{z}}_1, t + s : \dot{\mathbf{U}}, 0, 0) = \mathbf{b}(\mathbf{U}, \mathbf{z}_1, \dot{\mathbf{z}}_1, t) \tag{22}$$

where

$$\mathbf{b}(\mathbf{U}, \mathbf{z}_1, \dot{\mathbf{z}}_1, t) := \mathbf{A}_s\mathbf{K}(\mathbf{U}, \mathbf{z}_1, t) - D_s\mathbf{K}(\mathbf{U}, \mathbf{z}_1, t : 0, \dot{\mathbf{z}}_1, 1).$$

The forward difference approximation of the products of the Jacobians and vectors can be calculated with only an additional evaluation of the function, which requires notably less computational burden than an approximation of the Jacobians themselves.

Because (22) approximates a linear equation with respect to $\dot{\mathbf{U}}$, we can apply the GMRES algorithm [21] to (22). GMRES is a kind of Krylov subspace method for such a linear equation as $\mathbf{Ay} = \mathbf{b}$ where $\mathbf{A}$ is not necessarily symmetric or positive definite. GMRES at the kth iteration minimizes the residual $\rho := \|\mathbf{b} - \mathbf{Ay}\|$ with $\mathbf{y} \in \mathbf{y}_0 + \mathcal{K}_k$, where $\mathbf{y}_0$ is the initial guess and $\mathcal{K}_k$ denotes the Krylov subspace defined by $\mathcal{K}_k :=\text{span}\{\mathbf{r}_0, \mathbf{Ar}_0, \ldots, \mathbf{A}^{k-1}\mathbf{r}_0\}$ with $\mathbf{r}_0 := \mathbf{b} - \mathbf{Ay}_0$. GMRES also successively generates an orthonormal basis $\{\mathbf{v}_j\}_{j=1}^{k}$ for $\mathcal{K}_k$. Minimization is executed efficiently through the use of Givens rotations. In principle, GMRES reduces the residual monotonically and converges to the solution within the same number of iterations as the dimension of the equation. However, an important advantage of GMRES for a large linear equation is that a specified error tolerance, for example, $\rho \le \xi\|\mathbf{r}_0\|$ $(0 < \xi < 1)$, can be achieved with much fewer iterations. More detailed information about the implementation of C/GMRES algorithm is provided in [17].

V. Estimation Method

In this section, we consider the state estimation problem for the discrete nonlinear system model given by

$$\mathbf{z}_{j+1} = \mathbf{F}(\mathbf{z}_j, u_j) + \boldsymbol{\delta}_j. \tag{23}$$

Therein, $\mathbf{F} \in \mathbb{R}^M$ is given by $\mathbf{F} = \mathbf{A}^{-1}(\mathbf{z}_j)\mathbf{B}(\mathbf{z}_j, \mathbf{u}_j)$, where $\mathbf{A}$ and $\mathbf{B}$ are defined in (16a). Let $\boldsymbol{\delta} \in \mathbb{R}^M$ denote the process noise, which can be caused by disturbances and modeling errors. The temperature on the surface of the strip can only be detected by a thermal sensing device. Then, the output $\mathbf{y} \in \mathbb{R}^2$ of system (23) is given as

$$\begin{aligned} \mathbf{y}_j &= \mathbf{L}(\mathbf{z}_j) + \boldsymbol{\xi}_j \\ \mathbf{L}(\mathbf{z}_j) &:= \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 1 \end{bmatrix} \mathbf{z}_j = \begin{bmatrix} z_{1,j} \\ z_{M,j} \end{bmatrix} \end{aligned} \tag{24}$$

where $\boldsymbol{\xi} \in \mathbb{R}^2$ denotes the observation noise. The optimal estimate in the minimum mean-squared error sense is given by the conditional mean. Let $\hat{\mathbf{z}}_{j|k}$ be the mean of $\hat{\mathbf{z}}_j$ conditioned on all of the observations up to time k, i.e., $\hat{\mathbf{z}}_{j|k} = \mathbb{E}\left[\hat{\mathbf{z}}_j|\mathbf{Y}^k\right]$, where $\mathbf{Y}^k := \{\mathbf{y}_1, \mathbf{y}_2, \ldots \mathbf{y}_k\}$. The covariance of this estimate is denoted by $\mathbf{Q}^z_{j|k}$. Let $\mathbf{Q}^\delta_j$ and $\mathbf{Q}^\xi_j$ be the covariances of $\boldsymbol{\delta}_j$ and $\boldsymbol{\xi}_j$, respectively. It is assumed that the means of $\boldsymbol{\delta}_j$ and $\boldsymbol{\xi}_j$ are zero. The UKF [18] first predicts the mean and covariance of a future state using the process model and weighted sigma points

$$\boldsymbol{\chi}_{j+1|j} = \mathbf{F}(\boldsymbol{\chi}_j, u_j) \tag{25}$$

$$\hat{\mathbf{z}}_{j+1|j} = \sum_{i=0}^{2M} W^i \chi^i_{j+1|j} \tag{26}$$

$$\begin{aligned} \mathbf{Q}^z_{j+1|j} = &\sum_{i=0}^{2M} W^i \left(\chi^i_{j+1|j} - \hat{\mathbf{z}}_{j+1|j}\right)\left(\chi^i_{j+1|j} - \hat{\mathbf{z}}_{j+1|j}\right)' \\ &+ \mathbf{Q}^\delta_{j+1} \end{aligned} \tag{27}$$

where W^i and χ^i denote the weight and sigma point, respectively. The definitions of W^i, χ^i, and $\boldsymbol{\chi}$ can be found in [18]. After we redraw a new set of sigma points $\tilde{\chi}^i$ to incorporate the effect of the additive process noise, the predicted observation is calculated by

$$\hat{\mathbf{y}}_{j+1|j} = \sum_{i=0}^{2M} W^i \mathbf{L}\left(\tilde{\chi}^i_{j+1|j}\right). \tag{28}$$

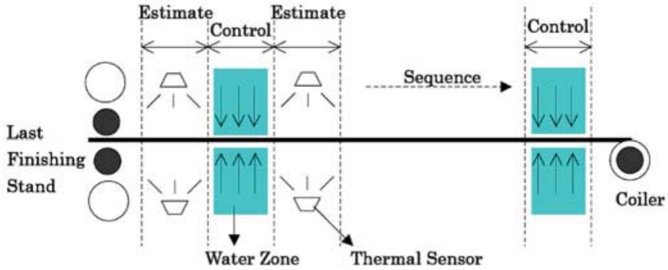

Fig. 2. Layout of the cooling process.

Moreover, the cross covariance $\mathbf{P}$ and innovation covariance $\mathbf{R}$ are determined by

$$\mathbf{P}_{j+1|j} = \sum_{i=0}^{2M} W^i \left(\tilde{\chi}^i_{j+1|j} - \hat{\mathbf{z}}_{j+1|j}\right) \times \left(\mathbf{L}\left(\tilde{\chi}^i_{j+1|j}\right) - \hat{\mathbf{y}}_{j+1|j}\right)' \tag{29}$$

$$\mathbf{R}_{j+1|j} = \mathbf{Q}^{\xi}_{j+1} + \sum_{i=0}^{2M} W^i \left(\mathbf{L}\left(\tilde{\chi}^i_{j+1|j}\right) - \hat{\mathbf{y}}_{j+1|j}\right) \times \left(\mathbf{L}\left(\tilde{\chi}^i_{j+1|j}\right) - \hat{\mathbf{y}}_{j+1|j}\right)'. \tag{30}$$

Consequently, the estimate at time $j+1$ is obtained by updating the prediction by the linear update rule

$$\mathbf{G}_{j+1} = \mathbf{P}_{j+1|j}\mathbf{R}^{-1}_{j+1|j} \tag{31a}$$

$$\hat{\mathbf{z}}_{j+1|j+1} = \hat{\mathbf{z}}_{j+1|j} + \mathbf{G}_{j+1}\left(\mathbf{y}_{j+1} - \hat{\mathbf{y}}_{j+1|j}\right) \tag{31b}$$

$$\mathbf{Q}^{z}_{j+1|j+1} = \mathbf{Q}^{z}_{j+1|j} - \mathbf{G}_{j+1}\mathbf{R}_{j+1|j}\mathbf{G}'_{j+1}. \tag{31c}$$

Note that a UKF is more accurate than an extended Kalman filter (EKF) [22] and easier to implement than an EKF, because a UKF does not involve any linearization steps, eliminating the need to derive of the Jacobian matrix of $\mathbf{F}$.

VI. Numerical Simulation

In this section, we consider the cooling process of a hot strip mill that can be regarded as a sequence of estimation zones and control zones. Fig. 2 shows that the cooling process on the ROT is divided into the estimation zone and the control zone. For simplicity, the velocity of the strip is fixed at 3 m/s and the length of each estimation zone and control zone is set as 6 m. A simulation of a sequence consisting of four estimation zones, each followed by a control zone, is performed here.

Here, $k(z,u)$ and $u(\omega, z_w)$ are given by

$$k(z,u) = \frac{u(\omega, z_w)}{b(z)} + \frac{\sigma\varepsilon(z^2 - z_a^2)^2}{b(z)} \tag{32}$$

$$u(\omega, z_w) = \theta_1 \omega^{\theta_2} z_w^{\theta_3} \tag{33}$$

where $\theta_1 = 4.59 \times 10^6$, $\theta_2 = 0.7$, $\theta_3 = -0.54$, $z_w = 10$, $z_a = 20$, $\sigma = 5.67 \times 10^{-8}$, and $\varepsilon = 0.8$. The first and second terms of $k(z,u)$ are adduced from [3] and [13], respectively. The specific heat capacity $c(z)$ and specific enthalpy $d(z)$ used in the simulation are shown in Fig. 3. The thermal conductivity $b(z)$ and desired temperature profile $z_f(t)$ used in the simulation are shown in Figs. 4 and 5, respectively. The initial states $z_0(x)$ and $\hat{z}_0(x)$ are set as $z_0(x) = 50\sin(\pi x/h) + 750$ and $\hat{z}_0(x) = 800$, respectively. $\boldsymbol{\delta}_j$ and $\boldsymbol{\xi}_j$ are set as zero-mean Gaussian noises with covariances $\mathbf{Q}^{\delta}_j = 0.2\mathbf{I}$ and $\mathbf{Q}^{\xi}_j = 0.2\mathbf{I}$, respectively. Other parameters are chosen as follows: $\Delta t = 5$ ms, $h = 0.01$, $M = 40$, $N = 5$, $\rho = 7860$, $[p,q,r,s] = [1, 10^{14}, 1, 1]$, $\omega_{\max} = 1$, and $u_{\max} = \theta_1 \omega_{\max}^{\theta_2} z_w^{\theta_3}$. The results of the simulation by the proposed method are shown in Figs. 6–10. Figs. 6 and 7 show the time histories of the real temperature $z(x,t)$ and estimated temperature $\hat{z}(x,t)$, respectively. Figs. 8 and 9 show the time

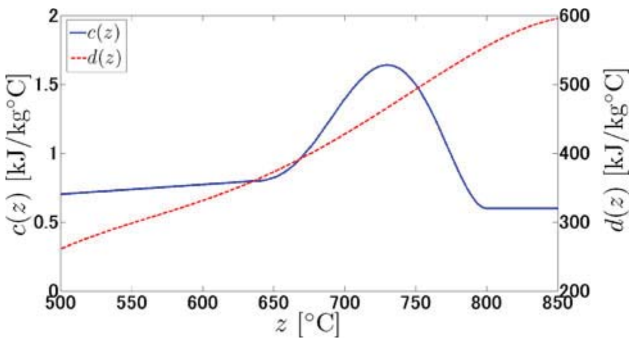

Fig. 3. $c(z)$ and $d(z)$ adduced from [14].

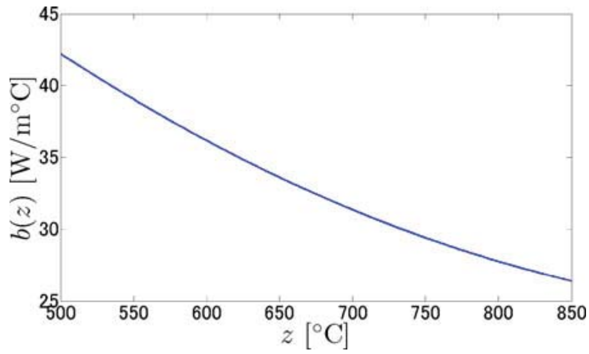

Fig. 4. $b(z)$ referred to from [23].

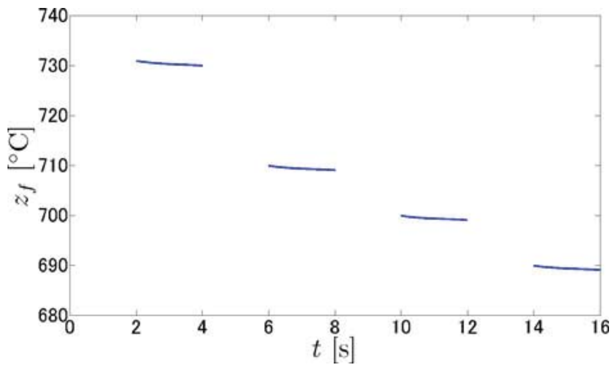

Fig. 5. Desired temperature profile z_f.

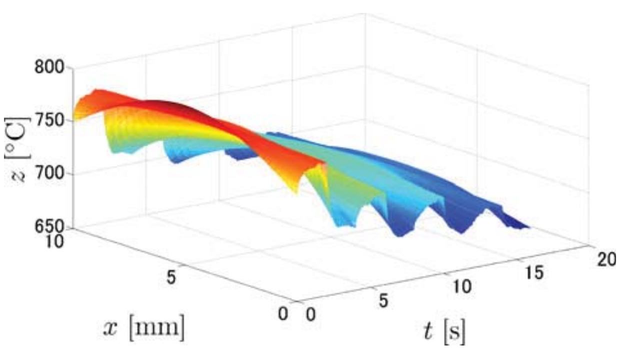

Fig. 6. Time history of the real temperature z.

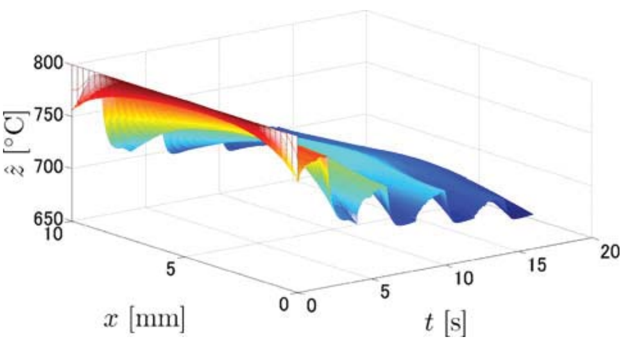

Fig. 7. Time history of the estimated temperature $\hat{z}$.

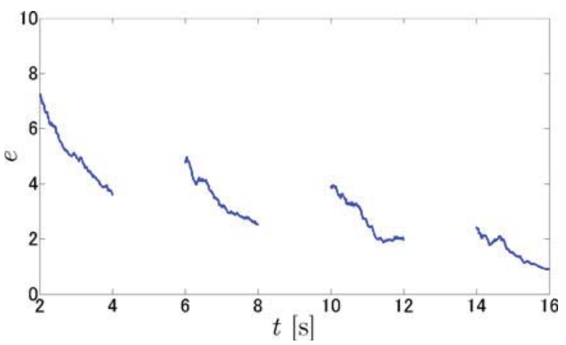

Fig. 8. Time history of the control error e.

histories of the average control error $e(t) := \|\mathbf{z}_f(t) - \mathbf{z}(t)\|/M$ in each control zone and the average estimation error $\hat{e}(t) := \|\mathbf{z}(t) - \hat{\mathbf{z}}(t)\|/M$ in each estimation zone, respectively. It can be seen that the temperature of the strip is successfully controlled and estimated by the proposed method. Fig. 10 shows the time history of the controlled water rate. It can be seen that the optimal strategy is reflected in the water consumption. The average computational times per time step for the controller and estimator are 26.9 and 13.8 ms, respectively. It is found that fast optimization can be achieved by the proposed method. The simulation was performed on a laptop computer (Panasonic CF-S9, CPU: Intel(R) Core(TM) i5, 2.4 GHz Memory: 3.4 GB OS: Windows 7, 32 bit Software: MATLAB).

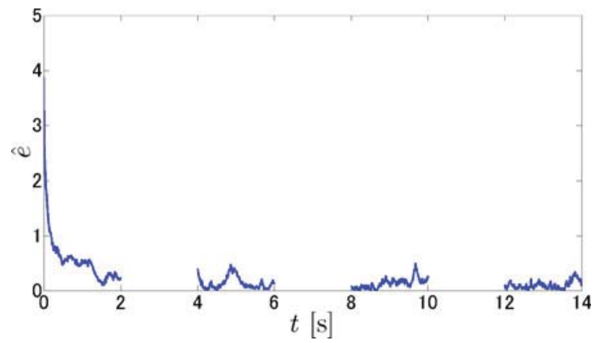

Fig. 9. Time history of the estimation error $\hat{e}$.

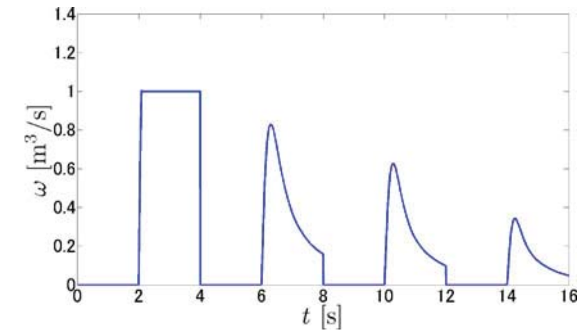

Fig. 10. Time history of the water flow rate ω.

VII. Conclusion

We have investigated the control and estimation problem for a hot strip mill cooling system. We first presented a design method of receding horizon control, in which the control performance over a finite future is optimized, for the temperature of a strip whose mathematical model is described by a nonlinear PDE. We adopted the C/GMRES method to solve the optimization problem, because the C/GMRES algorithm is not only faster but also more numerically robust than the conventional algorithm. Thus, an efficient algorithm for solving the optimal control problem for a class of nonlinear PDEs was established herein. Next, we designed an observer with a UKF for estimating the inhomogeneously distributed temperature of the strip. Taking the process noise and observation noise into consideration, the proposed observer is incorporated into the receding horizon controller. Therefore, the proposed method enables us to manage the water consumption and the robustness against modeling errors and disturbances. Finally, the effectiveness of the proposed method was verified by numerical simulation. The stability of the closed-loop system controlled by receding horizon control with a UKF is not theoretically guaranteed. Robust stability against modeling errors and disturbances should be addressed in future research.

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